



# Layered superconductors as negative-refractive-index metamaterials

## Citation

Rakhmanov, A. L., V. A. Yampol'skii, J. A. Fan, Federico Capasso, and Franco Nori. 2010. Layered superconductors as negative-refractive-index metamaterials. *Physical Review B* 81 (7): 10.1103.

## Published Version

doi:10.1103/PhysRevB.81.075101

## Permanent link

<http://nrs.harvard.edu/urn-3:HUL.InstRepos:11693947>

## Terms of Use

This article was downloaded from Harvard University's DASH repository, and is made available under the terms and conditions applicable to Other Posted Material, as set forth at <http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#LAA>

## Share Your Story

The Harvard community has made this article openly available.  
Please share how this access benefits you. [Submit a story](#).

[Accessibility](#)

## Layered superconductors as negative-refractive-index metamaterials

A. L. Rakhmanov,<sup>1,2</sup> V. A. Yampol'skii,<sup>1,3</sup> J. A. Fan,<sup>4</sup> Federico Capasso,<sup>4</sup> and Franco Nori<sup>1,5</sup><sup>1</sup>Advanced Science Institute, The Institute of Physical and Chemical Research (RIKEN), Saitama 351-0198, Japan<sup>2</sup>Institute for Theoretical and Applied Electrodynamics, Russian Academy of Science, 125412 Moscow, Russia<sup>3</sup>A. Ya. Usikov Institute for Radiophysics and Electronics, Ukrainian Academy of Science, 61085 Kharkov, Ukraine<sup>4</sup>School of Engineering and Applied Sciences, Harvard University, Cambridge, Massachusetts 02138, USA<sup>5</sup>Department of Physics, University of Michigan, Ann Arbor, Michigan 48109-1040, USA

(Received 21 December 2009; published 1 February 2010)

We analyze the use of layered superconductors as strongly anisotropic metamaterials, which can possess negative-refractive-index in a wide frequency range. Superconductors are of particular interest because they have the potential to support low losses, which is critical for applications such as super-resolution imaging. We show that low- $T_c$  ( $s$ -wave) superconductors can be used to construct layered heterostructures with *low losses* for  $T \ll T_c$ . However, the real part of their in-plane effective permittivity is very large, making coupling into the structure difficult. Moreover, even at low temperatures, layered high- $T_c$  superconductors have a large in-plane normal conductivity, producing large losses (due to  $d$ -wave symmetry). Therefore, it is difficult to enhance the evanescent modes in either low- $T_c$  or high- $T_c$  superconductors.

DOI: 10.1103/PhysRevB.81.075101

PACS number(s): 78.20.Ci, 42.25.Bs, 74.25.N-

## I. INTRODUCTION

Metamaterials are attracting considerable attention because of their unusual interaction with electromagnetic waves (see, e.g., Refs. 1–4). In particular, metamaterials supporting negative-refractive-index have the potential for sub-wavelength resolution<sup>5</sup> and aberration-free imaging.

## A. Limitations of standard metamaterials

The first proposed negative index metamaterials used sub-wavelength electric and magnetic structures to achieve simultaneously negative permittivity  $\epsilon$  and permeability  $\mu$  (see, e.g., Ref. 6). However, these “double negative” structures require intricate design and demanding fabrication techniques, are not “very subwavelength” and suffer from significant spatial dispersion effects. Moreover, the implicit overlapping electric and magnetic resonances (see, e.g., Refs. 7–10) often leads to resonant losses that, together with material losses, lead to significant degradation in metamaterial functionality. For example, small amounts of loss can substantially degrade subwavelength resolution. This manifestation of loss can be quantified by examining the figure of merit (FOM) in such materials, which is defined as  $|n'|/n''$  where  $n'$  and  $n''$  are the real and imaginary parts of the refractive index  $n$ , respectively. The FOMs of negative index materials in the visible and near-IR have experimentally ranged<sup>6,11</sup> from 0.1 up to 3.5.

## B. Metamaterials from strongly anisotropic compounds

Another promising route to creating metamaterials is to construct strongly anisotropic media; in particular, uniaxial anisotropic materials with different signs of the permittivity tensor components along,  $\epsilon_{\parallel}$ , and transverse,  $\epsilon_{\perp}$ , to the surface (see, e.g., Refs. 12–15). These materials can exhibit either: (i) positive refraction with; however, negative-refractive-index (when the *normal* to the sample surface components of the wave vector and the Poynting vector have

different signs); or (ii) negative refraction with positive refractive index (when the *parallel* to the sample surface components of the wave vector and the Poynting vector have different signs). Both cases can support super-resolution imaging.<sup>16,17</sup> These materials have also been proposed as a model system for scattering-free plasmonic optics<sup>18</sup> and subwavelength-scale waveguiding.<sup>19</sup> This route to metamaterials is particularly attractive because they are relatively straightforward to fabricate compared to double negative metamaterials, can support broadband negative-refractive-index or negative refraction, and do not require negative permeability and consequently suffer no magnetic resonance losses. The FOMs for such materials have been calculated<sup>12,20</sup> to be significantly greater than those measured in double negative materials.

Experimental schemes for creating strongly anisotropic uniaxial materials have typically involved the fabrication of subwavelength stacks of materials whose layers comprise alternating signs of permittivity. For example, alternating stacks of Ag and  $\text{Al}_2\text{O}_3$  (Ref. 17) and of doped and undoped semiconductors<sup>20</sup> have been demonstrated to support strong anisotropy in the visible and infrared frequency ranges respectively. However, spatial dispersion can strongly modify the optical response of the system relative to the ideal effective medium limit response;<sup>21</sup> strong local field oscillations exist even in the limit of  $l \ll a_0$ , where  $l$  is the length scale of the thin films in the material and  $a_0$  is the free-space electromagnetic wavelength. This imposes limitations to subwavelength imaging and waveguiding in such materials. Spatial dispersion, i.e., the dependence of the dielectric function on both  $\omega$  and  $\mathbf{k}$ , can be reduced by making composite structures with thinner layers. However, there exist practical material deposition limitations to thin-film stacks, and the minimum film thickness is set by damping due to electron scattering at the thin-film interface and is approximately  $a_0 v_F / c \sim a_0 / 100$  where  $v_F$  is the Fermi velocity in the material.<sup>22</sup> These composite structures are limited in practice as “ideal” strongly anisotropic materials.

We analyze here the idea of using superconductors as metamaterials (see, e.g., Refs. 23–28). In particular, we consider layered cuprate superconductors<sup>28</sup> and artificial superconducting-insulator systems<sup>29</sup> as candidates for strongly anisotropic metamaterials. Unlike the composite structures discussed earlier, layered superconductors are not limited in performance by the spatial dispersion effects discussed in Ref. 21. Note here, that layered superconducting materials obey a number of interesting and important optical properties in the THz frequency range (see, e.g., Refs. 30–32).

We will analyze these materials in the specific context of subwavelength resolution, which can be achieved by the amplification of evanescent waves.<sup>5</sup> This amplification is high when  $n$  is close to unity and its imaginary part is small.<sup>5,33</sup> For the incident  $p$ -polarized waves considered here, subwavelength resolution requires

$$\text{Im}(\varepsilon) \ll \exp(-2k_{\perp}L),$$

where  $k_{\perp}$  is the wave-vector component perpendicular to the surface, and  $L$  is the plane lens thickness.<sup>33</sup> For evanescent modes with  $k_{\perp} = 2\omega/c = 2k_0$  and  $L/a_0 = 0.1$ , we have  $\text{Im}(\varepsilon) \ll 0.081$ .

We show that in the case of natural high- $T_c$  cuprates the losses are high at any reasonable frequency. In the case of artificial-layered structures prepared from low- $T_c$  superconductors, the losses can be reduced significantly at low temperatures,  $T \ll T_c$ , where  $T_c$  is the critical temperature. The frequency range for such a metamaterial is  $\hbar\omega < 2\Delta$ , where  $\Delta$  is the superconducting gap, which corresponds to a maximum frequency in the THz range for low- $T_c$  superconductors. We prove that the in-plane permittivity for low- $T_c$  multilayers is large, preventing the effective enhancement of evanescent waves. This is problematic because subwavelength resolution<sup>5</sup> requires the amplification of evanescent waves. Note that Refs. 25–27 only focus on the zero-frequency dc case.

## II. EFFECTIVE PERMITTIVITY

We study a medium consisting of a periodic stack of superconducting layers of thickness  $s$  and insulating layers of thickness  $d$  with Josephson coupling between successive superconducting planes. The number of layers is large,  $L/(s+d) = N \gg 1$  and  $s$  is smaller than: the in-plane magnetic field penetration depth  $\lambda_{\parallel}$ , transverse skin depth  $\delta_{\perp}(\omega)$ , and wavelength

$$a(\omega) \sim \frac{2\pi c}{\omega\sqrt{|\varepsilon(\omega)|}}.$$

We calculate the effective permittivity,  $\hat{\varepsilon} = (\varepsilon_{\parallel}, \varepsilon_{\perp})$ , of the layered system in the case of  $p$ -wave refraction.

Layered superconductors with Josephson couplings can be described by the Lawrence-Doniach model, where the averaged current components can be expressed as<sup>30</sup>

$$J_{\perp} = J_c \sin \varphi_n + \frac{\sigma_{\perp} \Phi_0 \dot{\varphi}_n}{2\pi c(s+d)},$$

$$J_{\parallel} = \frac{c\Phi_0 p_n}{8\pi^2 \lambda_{\parallel}^2} + \sigma_{\parallel} E_{\parallel}, \quad (1)$$

where  $\varphi_n$  is the gauge-invariant phase difference between the  $(n+1)$ th and  $n$ th superconducting layers,  $p_n$  is the in-plane superconducting momentum,

$$J_c = c\Phi_0/(8\pi^2 d\lambda_{\perp}^2),$$

is the transverse supercurrent density,  $\Phi_0$  is the magnetic flux quantum, and  $\lambda_{\perp}$  is the transverse magnetic field penetration depths. Also  $\sigma_{\perp}$  and  $\sigma_{\parallel}$  are the averaged transverse and in-plane quasiparticle conductivities. The transverse  $E_{\perp}$  and in-plane  $E_{\parallel}$  components of the electric field are related to the gauge-invariant phase difference and superconducting momentum by<sup>30,31</sup>

$$(1 - \alpha \nabla_n^2) E_{\perp} = \frac{\Phi_0}{2\pi c(s+d)} \dot{\varphi}_n, \quad E_{\parallel} = \frac{\Phi_0}{2\pi c} \dot{p}_n, \quad (2)$$

where  $\nabla_n^2 f(n) = f(n+1) + f(n-1) - 2f(n)$ ,  $\alpha = \varepsilon R_D^2/(sd)$  is the capacitive coupling between layers, and  $R_D$  is the Debye length. We linearize the first of Eq. (1) and consider a linear electromagnetic wave

$$E_{\parallel,\perp}(x, n, t) = \sum_q \int \frac{dk d\omega}{(2\pi)^2} E_{\parallel,\perp}(k, q, \omega) e^{(-i\omega t + ikx + iqn)},$$

where  $q = \pi l/(N+1)$ ,  $l = 0, \pm 1, \pm 2$ , and the  $x$  axis is in the plane of the layers. Using Eqs. (1) and (2), we obtain

$$\frac{J_{\perp}}{E_{\perp}} = (1 + \alpha \tilde{q}^2) \left[ \sigma_{\perp} - \frac{\varepsilon \omega_p^2 (s+d)}{4\pi i d \omega} \right],$$

$$\frac{J_{\parallel}}{E_{\parallel}} = \sigma_{\parallel} - \frac{\varepsilon \gamma^2 \omega_p^2}{4\pi i \omega}, \quad (3)$$

where

$$\omega_p = c/(\lambda_{\perp} \sqrt{\varepsilon}),$$

is the Josephson plasma frequency,  $\varepsilon$  is the interlayer permittivity,  $\gamma = \lambda_{\perp}/\lambda_{\parallel}$ , and  $\tilde{q}^2 = 2(1 - \cos q)$ . Averaged over the sample volume, the Maxwell equation has the form

$$c \nabla \times \mathbf{H} = 4\pi \mathbf{J} + \partial \mathbf{D} / \partial t,$$

where  $D_{\parallel} = \varepsilon_{\parallel}^0 E_{\parallel}$  and  $D_{\perp} = \varepsilon_{\perp}^0 E_{\perp}$ . In the effective medium approximation, the components of the permittivity tensor can be expressed as<sup>34</sup>

$$\varepsilon_{\parallel}^0 = \frac{d\varepsilon + s}{s+d}, \quad \varepsilon_{\perp}^0 = \frac{\varepsilon(s+d)}{s\varepsilon + d},$$

where we assume that  $\varepsilon_{\text{superconductor}} = 1$ . Fourier transforming the above Maxwell equation, we derive

$$c[\nabla \times \mathbf{H}]_{\perp}(k, q, \omega) = -\varepsilon_{\perp} \dot{E}_{\perp},$$

$$c[\nabla \times \mathbf{H}]_{\parallel}(k, q, \omega) = -\varepsilon_{\parallel} \dot{E}_{\parallel}, \quad (4)$$

where

$$\varepsilon_{\parallel} = \varepsilon_{\parallel}^0 - \frac{4\pi J_{\parallel}}{i\omega E_{\parallel}}, \quad \varepsilon_{\perp} = \varepsilon_{\perp}^0 - \frac{4\pi J_{\perp}}{i\omega E_{\perp}}.$$

Therefore, we finally obtain

$$\varepsilon_{\perp} = \varepsilon_{\perp}^0 - \frac{4\pi(1 + \alpha\tilde{q}^2)\sigma_{\perp}}{i\omega} - \varepsilon(1 + \alpha\tilde{q}^2)\frac{\omega_p^2(s+d)}{\omega^2 d},$$

$$\varepsilon_{\parallel} = \varepsilon_{\parallel}^0 - \frac{4\pi\sigma_{\parallel}}{i\omega} - \varepsilon\gamma^2\frac{\omega_p^2}{\omega^2}. \quad (5)$$

Thus,  $\varepsilon_{\parallel} < 0$  and  $\varepsilon_{\perp} > 0$  in the frequency range

$$\sqrt{(1 + \alpha\tilde{q}^2)\frac{s\varepsilon + d}{d}} < \frac{\omega}{\omega_p} < \gamma\sqrt{\frac{\varepsilon(s+d)}{d\varepsilon + s}}. \quad (6)$$

If the incident angle is close to normal and anisotropy is large,  $\gamma \gg 1$ , we can find an estimate

$$\text{FOM} \approx 2 \left| \frac{\text{Re}(\varepsilon_{\parallel})}{\text{Im}(\varepsilon_{\parallel})} \right| \approx \frac{\varepsilon\gamma^2\omega^3}{2\pi\sigma_{\parallel}\omega_p^2}.$$

Electromagnetic waves propagate in the layered superconductors if  $\omega > \omega_p$ . Thus, the results obtained are valid if

$$\omega_p < \omega < \omega_c = 2\Delta/\hbar.$$

Below we analyze separately the different cases of a typical high- $T_c$  layered superconductor,  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  (Bi2212), and also an artificial low- $T_c$  layered structure made from Nb.

### III. LAYERED HIGH- $T_c$ SUPERCONDUCTORS

In the case of Bi2212, it is known that  $s \ll d$  since  $s \approx 0.2$  nm while  $d = 1-2$  nm,  $\varepsilon = 12$ ,  $\alpha \approx 0.1$ , and at low-temperatures ( $T \ll T_c = 90$  K)  $\omega_p \approx 10^{12}$  s $^{-1}$ ,  $\gamma = 500$ ,  $\sigma_{\parallel} \approx 4 \times 10^4$   $\Omega^{-1}$  cm $^{-1}$ , and  $\sigma_{\perp} \approx 2 \times 10^{-3}$   $\Omega^{-1}$  cm $^{-1}$  (see, e.g., Refs. 30 and 35). In this case, Eq. (5) can be rewritten as

$$\varepsilon_{\perp} \approx \varepsilon \left( 1 - \frac{\omega^2}{\omega_p^2} \right) + \frac{4\pi i \sigma_{\perp}}{\omega}, \quad \varepsilon_{\parallel} \approx \varepsilon \left( 1 - \frac{\gamma^2 \omega^2}{\omega_p^2} \right) + \frac{4\pi i \sigma_{\parallel}}{\omega}.$$

The calculated frequency dependence of the permittivity for Bi2212 is shown in Figs. 1 and 2. The superconducting gap for Bi2212 is estimated as  $\Delta \approx 2-3k_B T_c$ , with

$$\omega_c \approx 5 \times 10^{13} \text{ s}^{-1} \ll \gamma\omega_p.$$

Thus, for any incident angle, Bi2212 has negative  $n$  in the frequency range from about 0.15 to 7.5 THz, or in the wavelength domain  $40 \mu\text{m} \leq a \leq 2$  mm. However, the use of Bi2212 as metamaterial has a disadvantage since the in-plane quasiparticle conductivity  $\sigma_{\parallel}$  is large, even at helium temperatures, see Fig. 3. As it is seen from Fig. 3,  $\sigma_{\parallel} \neq 0$  when  $T \rightarrow 0$ , which is typical for superconductors having a  $d$ -type symmetry of the order parameter. In addition, the usual dimensions of high-quality Bi2212 single crystals are less than 1 mm in the in-plane direction and about 30–100  $\mu\text{m}$  in the transverse direction. Thus, it might be difficult to use Bi2212 single crystals as metamaterials, or elements of a superlens.

### IV. LOW- $T_c$ ARTIFICIAL LAYERED STRUCTURES

The thickness of the insulator in Josephson junctions is about a few nm. To attain a low-loss regime and reach the

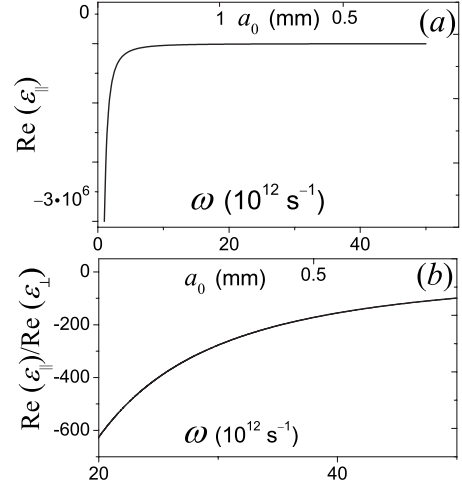


FIG. 1. Dependence of the real part of the permittivity  $\hat{\varepsilon}$  in Bi2212 on the frequency  $\omega$  (or wavelength  $a_0$ ), calculated from Eq. (5): (a) real part of the in-plane permittivity  $\varepsilon_{\parallel}(\omega)$ ; (b) ratio of the real parts of the in-plane and transverse permittivities.

bulk critical temperature, the thickness of the superconducting layers should be larger or about the superconductor coherence length  $\xi$ . For clean superconductors,  $\xi$  is about tens of nm. Thus, for low- $T_c$  artificial-layered structures, it is reasonable to analyze the case  $d \ll s$ . In this limit,

$$\lambda_{\parallel} = \lambda \sqrt{(s+d)/s} \approx \lambda,$$

where  $\lambda$  is the bulk magnetic field penetration depth and

$$\alpha = \varepsilon R_D^2/(sd) \ll 1$$

in any realistic case. It is easy to choose an insulator with very low conductivity  $\sigma_i$  to satisfy the condition  $\sigma_i \ll \sigma_s d/s$

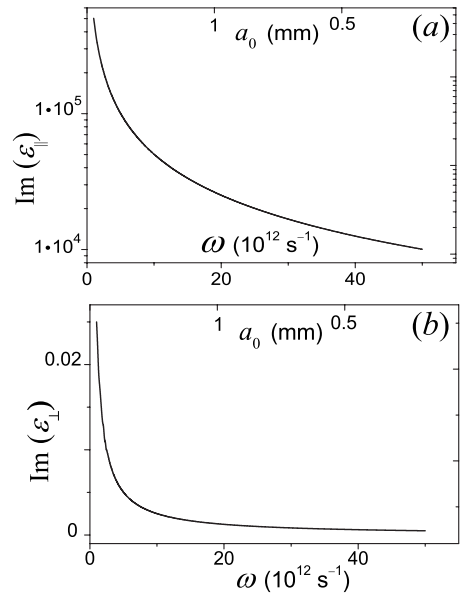


FIG. 2. Dependence of the imaginary part of the permittivity  $\hat{\varepsilon}$  in Bi2212 on the frequency  $\omega$  (or wavelength  $a_0$ ), calculated from Eq. (5): (a) imaginary part of the in-plane permittivity  $\varepsilon_{\parallel}(\omega)$ ; (b) imaginary part of the transverse permittivity  $\varepsilon_{\perp}(\omega)$ .

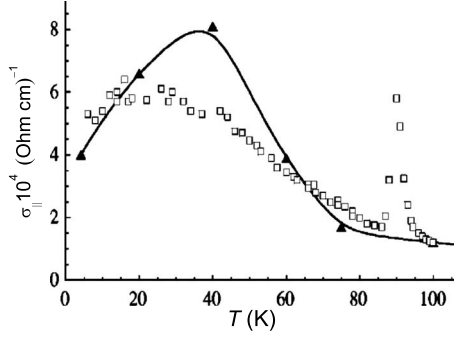


FIG. 3. Temperature dependence of the in-plane quasiparticle conductivity  $\sigma_{\parallel}(T)$  in Bi2212; solid triangles are low frequency data from Ref. 35, open squares correspond to 14.4 GHz data from Refs. 36 and 37.

at any reasonable temperature, where  $\sigma_s$  is the quasiparticle conductivity of the superconductor. In this case we have

$$\varepsilon_{\perp}^0 = 1, \quad \varepsilon_{\parallel}^0 = 1, \quad \sigma_{\perp} = \frac{\sigma_s}{d}, \quad \sigma_{\parallel} = \sigma_s.$$

Equations (5) for the effective permittivity can now be rewritten as

$$\varepsilon_{\perp} \approx \left(1 - \varepsilon \frac{s\omega_p^2}{d\omega^2}\right) + \frac{4\pi i\sigma_s}{\omega d}, \quad \varepsilon_{\parallel} \approx \varepsilon \left(1 - \gamma^2 \frac{\omega_p^2}{\omega^2}\right) + \frac{4\pi i\sigma_s}{\omega}. \quad (7)$$

Therefore, the refraction index  $n$  is negative if

$$\sqrt{\frac{\varepsilon s}{d}} < \frac{\omega}{\omega_p} < \gamma. \quad (8)$$

For artificial structures,  $\gamma$  can be easily made of the order of, or even much larger than, in natural layered superconductors. In contrast to  $d$ -wave high- $T_c$  superconductors, for bulk  $s$ -wave superconductors, the quasiparticle conductivity  $\sigma_s$  tends to zero for decreasing  $T$ . Thus, in principle, the imaginary part of  $\varepsilon_{\parallel}$  could be made as small as necessary by cooling the system.

Consider now Nb superconducting layers. For estimates we can take:<sup>38</sup>  $T_c=9.3$  K,  $\lambda(T=0)=44$  nm,  $\xi=38$  nm, electron mean-free-path  $l_e=20$  nm, and normal state conductivity  $\sigma_n=0.85 \times 10^6 \Omega^{-1} \text{cm}^{-1}$ . Thus, a reasonable thickness for the superconducting layers can be chosen as

$$s = 30 - 40 \text{ nm} \ll a(\omega_c) \gtrsim 100 - 200 \text{ nm}.$$

The superconducting properties of Nb are well described in the BCS weak-coupling approximation.<sup>38</sup> In particular, its conductivity  $\sigma_s(\omega, T)$  can be calculated using the Mattis-Bardeen theory<sup>39</sup> (Fig. 4). At low temperatures,  $T \ll T_c$ , in the weak-coupling BCS limit, we have  $\Delta=1.76k_B T_c$ . When  $\omega < \omega_c$  and  $T \ll T_c$ , we can rewrite the Mattis-Bardeen formula for conductivity<sup>38,39</sup> in the form

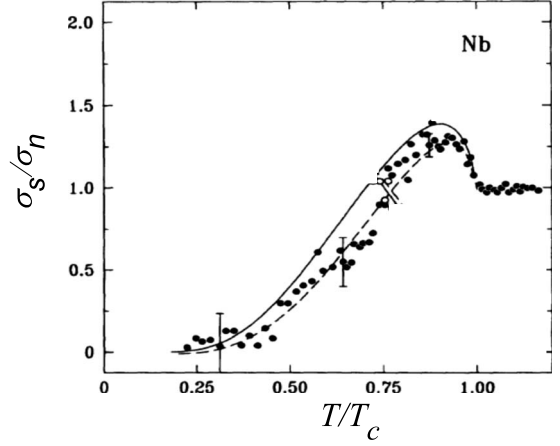


FIG. 4. The dependence (Ref. 38) of  $\sigma_s/\sigma_n$  on  $t \equiv T/T_c$ ; points: experimental data for Nb at about 60 GHz; solid line: Mattis-Bardeen theory in the weak-coupling BCS limit; dashed line: strong-coupling Eliashberg prediction. (Ref. 38)

$$\frac{\sigma_s}{\sigma_n} = \left[ 1 - \exp\left(-\frac{3.52 \omega}{\omega_c t}\right) \right] \frac{\omega_c}{\omega} \times \int_1^{\infty} \frac{(u^2 + 1 + 2u\omega/\omega_c) \exp\left(-\frac{1.76u}{t}\right)}{\sqrt{(u^2 - 1)[(u + 2\omega/\omega_c)^2 - 1]}} du, \quad (9)$$

where  $t=T/T_c$ . The results of our calculations are shown in Fig. 5. These calculations demonstrate that the losses in artificial structures made from low- $T_c$  superconductors *can be extremely low*. The maximum frequency  $\omega_c=3.52k_B T_c/\hbar$  for Nb corresponds to approximately 0.7 THz. From the results presented in Fig. 5, we can estimate that at  $\omega \sim \omega_c$  the imaginary part of  $\varepsilon_{\parallel}$  is lower than  $10^{-3}$  if  $T < 1$  K. At higher frequencies,  $\omega > \omega_c$ , the conductivity of the superconductor is about the conductivity of the normal metal and it cannot be easily used as a metamaterial with low losses.

Note also that by an appropriate choice of insulator,  $s$ , and  $d$ , we can vary the parameters  $\gamma$  and  $\omega_p$  in a wide range. If we assume that  $\varepsilon \sim 10$ , then to fulfill conditions (8) for  $\omega_p < \omega_c$  we should prepare highly anisotropic heterostructures

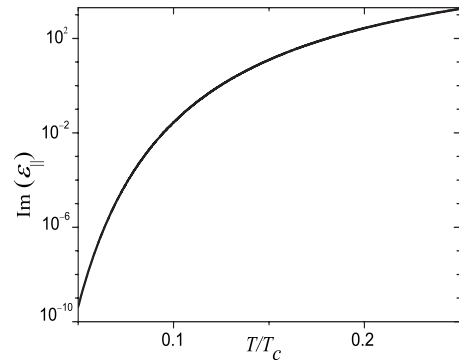


FIG. 5. Calculated, from Eqs. (7) and (9), temperature dependence of the imaginary part of  $\varepsilon_{\parallel}(t)=\varepsilon_{\parallel}(T/T_c)$ , for a Nb-based layered structure, with  $\omega=0.9\omega_c$ ,  $\varepsilon=10$ ,  $s/d=5$ , and  $\gamma=500$ ; here:  $\omega_p/\omega_c=0.1$ ,  $\text{Re}(\varepsilon_{\perp})=0.393$ , and  $\text{Re}(\varepsilon_{\parallel}) \approx -3 \times 10^6$ .



with  $\gamma > 10^3$ . If the anisotropy is large, we can find from Eq. (7) that

$$\text{Re}(\varepsilon_{\parallel}) \approx -c^2/\lambda^2\omega^2.$$

The absolute value of  $\text{Re}(\varepsilon_{\parallel})$  is very large,

$$|\text{Re}(\varepsilon_{\parallel})| \geq c^2/\lambda^2\omega_c^2 \approx 3 \times 10^6.$$

These estimates suggest that low- $T_c$  superconducting multilayers might not work as practical metamaterials.

The metamaterial properties of layered superconductors, either natural or artificial, can be tuned varying the temperature or an in-plane magnetic field, which strongly affects the transverse critical current density and, consequently, the plasma frequency. But applying a magnetic field increases dissipation, which is undesirable. Note also that the estimates made above show that the FOM may be very large for the systems considered here, however, this does not mean necessarily that these media can be easily used as practical metamaterials.

## V. CUPRATES IN THE NORMAL STATE

There is experimental evidence that cuprate superconductors have strongly anisotropic optical characteristics in the normal state.<sup>40,41</sup> For example, it was observed that  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  supports negative permittivity along the CuO planes at frequencies up to the mid- and near-IR range.<sup>40</sup> Moreover, these optical properties could be finely tuned by varying the stoichiometry. Such natural materials are thus candidates for practical anisotropic metamaterials. The use of cuprates in the normal state has evident advantages, such as

operating above  $\omega_c$  and to work at room temperature. However, the normal conductivity of cuprates is of the same order as their quasiparticle conductivity in the superconducting state (see, e.g., Fig. 3 and Ref. 35). The metamaterial properties of cuprates in the normal state require a separate analysis and will be performed elsewhere.

## VI. CONCLUSIONS

Here we analyze the properties of anisotropic metamaterials made from layered superconductors. We show that these materials can have a negative-refractive-index in a wide frequency range for arbitrary incident angles. However, superconducting metamaterials made from natural layered high- $T_c$  cuprates have a large in-plane normal conductivity, even at very low temperatures, due to  $d$ -wave symmetry of their superconducting order parameter. Therefore, these are very lossy. Nevertheless, low- $T_c$   $s$ -wave superconductors allow to produce metamaterials with *low losses* at low temperatures,  $T \ll T_c$ . But the real part of their in-plane permittivity is very large, reducing the enhancement of the evanescent modes and potentially limiting the use of superconducting structures as practical metamaterials.

## ACKNOWLEDGMENTS

F.N. gratefully acknowledges partial support from the National Security Agency (NSA), Laboratory of Physical Sciences (LPS), Army Research Office (ARO), National Science Foundation (NSF) Grants No. EIA-0726909 and No. JSPS-RFBR 09-02-92114. F.C. gratefully acknowledges useful discussions with E. Narimanov.

- 
- <sup>1</sup>V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968).  
<sup>2</sup>J. B. Pendry and D. R. Smith, Phys. Today **57**(6), 37 (2004).  
<sup>3</sup>K. Y. Bliokh, Y. P. Bliokh, V. Freilikher, S. Savel'ev, and F. Nori, Rev. Mod. Phys. **80**, 1201 (2008).  
<sup>4</sup>A. V. Kats, S. Savel'ev, V. A. Yampol'skii, and F. Nori, Phys. Rev. Lett. **98**, 073901 (2007).  
<sup>5</sup>J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).  
<sup>6</sup>V. M. Shalaev, Nat. Photonics **1**, 41 (2007).  
<sup>7</sup>R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).  
<sup>8</sup>H. O. Moser, B. D. F. Casse, O. Wilhelmi, and B. T. Saw, Phys. Rev. Lett. **94**, 063901 (2005).  
<sup>9</sup>T. Koschny, M. Kafesaki, E. N. Economou, and C. M. Soukoulis, Phys. Rev. Lett. **93**, 107402 (2004).  
<sup>10</sup>S. Zhang, W. Fan, N. C. Panoiu, K. J. Malloy, R. M. Osgood, and S. R. J. Brueck, Phys. Rev. Lett. **95**, 137404 (2005).  
<sup>11</sup>J. Valentine, S. Zhang, T. Zentgraf, E. Ulin-Avila, D. A. Genov, G. Bartal, and X. Zhang, Nature (London) **455**, 376 (2008).  
<sup>12</sup>V. A. Podolskiy and E. E. Narimanov, Phys. Rev. B **71**, 201101(R) (2005).  
<sup>13</sup>J. B. Pendry, Science **306**, 1353 (2004).  
<sup>14</sup>A. Alu and N. Engheta, J. Opt. Soc. Am. B **23**, 571 (2006).  
<sup>15</sup>O. V. Ivanov and D. I. Sementsov, Crystallogr. Rep. **45**, 487 (2000).  
<sup>16</sup>Z. Jacob, L. V. Alekseyev, and E. Narimanov, Opt. Express **14**, 8247 (2006).  
<sup>17</sup>Z. Liu, H. Lee, Y. Xiong, C. Sun, and X. Zhang, Science **315**, 1686 (2007).  
<sup>18</sup>J. Elser and V. A. Podolskiy, Phys. Rev. Lett. **100**, 066402 (2008).  
<sup>19</sup>A. A. Govyadinov and V. A. Podolskiy, Phys. Rev. B **73**, 155108 (2006).  
<sup>20</sup>A. J. Hoffman, L. Alekseyev, S. S. Howard, R. J. Franz, D. Wasserman, V. A. Podolskiy, E. E. Narimanov, D. L. Sivco, and C. Gmachl, Nature Mater. **6**, 946 (2007).  
<sup>21</sup>J. Elser, V. A. Podolskiy, I. Salakhutdinov, and I. Avrutsky, Appl. Phys. Lett. **90**, 191109 (2007).  
<sup>22</sup>A. A. Govyadinov and V. A. Podolskiy, J. Mod. Opt. **53**, 2315 (2006).  
<sup>23</sup>M. Ricci, N. Orloff, and S. M. Anlage, Appl. Phys. Lett. **87**, 034102 (2005).  
<sup>24</sup>C. Du, H. Chen, and S. Li, Phys. Rev. B **74**, 113105 (2006).  
<sup>25</sup>B. Wood and J. B. Pendry, J. Phys.: Condens. Matter **19**, 076208 (2007).  
<sup>26</sup>F. Magnus, B. Wood, J. Moore, K. Morrison, G. Perkins, J. Fyson, M. C. K. Wiltshire, D. Caplin, L. F. Cohen, and J. B. Pendry, Nature Mater. **7**, 295 (2008).  
<sup>27</sup>E. Narimanov, Nature Mater. **7**, 273 (2008).

- <sup>28</sup>M. B. Romanowsky and F. Capasso, *Phys. Rev. A* **78**, 042110 (2008).
- <sup>29</sup>A. Pimenov, P. Przyslupski, A. Loidl, and B. Dabrowski, *Phys. Rev. Lett.* **95**, 247009 (2005).
- <sup>30</sup>A. E. Koshelev and I. Aranson, *Phys. Rev. B* **64**, 174508 (2001).
- <sup>31</sup>S. E. Savel'ev, V. A. Yampol'sky, A. L. Rakhmanov, and F. Nori, arXiv:0903.2969, *Rep. Prog. Phys.* (to be published).
- <sup>32</sup>S. Savel'ev, A. L. Rakhmanov, and F. Nori, *Phys. Rev. Lett.* **94**, 157004 (2005); S. Savel'ev, V. Yampol'skii, and F. Nori, *ibid.* **95**, 187002 (2005); V. A. Yampol'skii, A. V. Kats, M. L. Nesterov, A. Y. Nikitin, T. M. Slipchenko, S. Savel'ev, and F. Nori, *Phys. Rev. B* **76**, 224504 (2007); V. A. Yampol'skii, D. R. Gulovich, S. Savel'ev, and F. Nori, *ibid.* **78**, 054502 (2008); V. A. Yampol'skii, A. V. Kats, M. L. Nesterov, A. Y. Nikitin, T. M. Slipchenko, S. Savel'ev, and F. Nori, *ibid.* **79**, 214501 (2009).
- <sup>33</sup>N. Garcia and M. Nieto-Vesperinas, *Phys. Rev. Lett.* **88**, 207403 (2002).
- <sup>34</sup>L. D. Landau *et al.*, *Electrodynamics of Continuous Media* (Butterworth-Heinemann, Oxford, 1995).
- <sup>35</sup>Yu. I. Latyshev, A. E. Koshelev, and L. N. Bulaevskii, *Phys. Rev. B* **68**, 134504 (2003).
- <sup>36</sup>S.-F. Lee, D. C. Morgan, R. J. Ormeno, D. Broun, R. A. Doyle, J. R. Waldram, and K. Kadowaki, *Phys. Rev. Lett.* **77**, 735 (1996).
- <sup>37</sup>H. Kitano *et al.*, *J. Low Temp. Phys.* **117**, 1241 (1999).
- <sup>38</sup>O. Klein, E. J. Nicol, K. Holczer, and G. Grüner, *Phys. Rev. B* **50**, 6307 (1994).
- <sup>39</sup>D. C. Mattis and J. Bardeen, *Phys. Rev.* **111**, 412 (1958).
- <sup>40</sup>S. Uchida, T. Ido, H. Takagi, T. Arima, Y. Tokura, and S. Tajima, *Phys. Rev. B* **43**, 7942 (1991).
- <sup>41</sup>S. Tajima, G. D. Gu, S. Miyamoto, A. Odagawa, and N. Koshizuka, *Phys. Rev. B* **48**, 16164 (1993).